Bonus Maths 4: The 3/4/5bet Game Revisited

In the book, I discussed a way of approaching the 3/4/5bet game mathematically. The take home message was, as usual, that playing loose OOP is pretty disastrous, because it's easy to be auto-exploited by the value range of the player who is IP. Since then it has been pointed out to me that there have been other attempts to use game theoretical ideas to construct optimal ranges for the 3bet/4bet/5bet game, notably the work described in this online article by *bugs*, who says that it is based on a video by Matt Janda. Having read this article, I decided to take a look at how the ideas presented in it translate into the mathematical framework that I set up in the book. From a game theoretical point of view, even though the arguments used are plausible, there are some problems with them, and I will give my take on this below. I've also been able to come up with a different way to try to approximate optimal 3/4/5bet ranges, using some of the ideas on zero sum games that I've tried to explain in my previous articles. If you want to read that as 'JB has learned some more game theory since writing the book' then feel free to do so.

Before I start, I want to make a couple of things clear from the outset. First of all, I am a fish, and Mr Janda is clearly a decent poker player so, whatever the flaws that I might find in his analysis, I expect that the ranges he suggests work out well in practice. I bought his book, and I would recommend it to you – after you've read our book of course! Remember, you have to judge what I'm about to present on the basis of the analysis and the underlying assumptions that I make. Secondly, as I hope you've realised by now, NLHE is a vast game, and whenever we try to isolate some subgame, the 3/4/5bet game being a prime example, we have to make a judgement call on what we can and can't neglect in our model. This is true with any sort of mathematical modelling. Anybody who thinks that applied mathematics is a dry subject with little room for heated debate probably thinks that it's all projectiles and blocks sliding down planes. Modelling real, complicated systems isn't as straightforward, and I'd be very interested to get some feedback on this article from game theorists and poker players alike. With this in mind, I'll outline the content.

I'll begin by describing Mr Janda's argument (even though my source is the article by *bugs*, so I am implicitly assuming that he hasn't misrepresented Mr Janda), and relate it to the notation that I used in the book. I'll then point out what I perceive to be its flaws. Next, I'll show you how to modify the argument that I used in the book to convert the problem into a zero sum game. I'll then add an additional layer of complexity by not specifying each player's value range in advance, which is the main simplification that I used in the book, instead leaving them essentially undetermined. I'll then use standard techniques to find the Nash equilibrium for various opening and calling ranges, and try to interpret the results. Finally, I'll present some results from using a different approach, which, and this is a first for me, actually makes use of hand v hand equities and card removal effects, although, as we'll see, this extra realism does come with some extra computational costs.

Mr Janda's Argument, as Presented by *bugs*

I'll use the notation that I defined in the book, namely that Player 1, who is OOP, is prepared to stack off for 100bb with V_1 combos, 4bet bluffs and folds to a 5bet with B_4 combos, and folds F_1 combos to a 3bet, so that he has a total of $V_1 + B_4 + F_1$ combos in his opening range. Player 2, who is IP, is prepared to stack off for 100bb with V_2 value combos, 3bet bluffs and folds to a 4bet with B_3 combos, and 5bet bluffs with B_5 combos. The bet sizes used by Mr Janda are a little different to the sizes I used in the book – the opening raise is to 3.5bb, the 3bet is to 12bb and the 4bet is to 27bb – but this doesn't materially affect the arguments, which I will now summarise.

i) Player 1 needs to stop Player 2 from making an instant profit with his 3bet bluff of 12bb into a pot of 5bb, so he must 4bet with 5/(5+12) = 29.4% of his range. In my notation,

$$\frac{B_4 + V_1}{B_4 + V_1 + F_1} = \frac{5}{5 + 12}.$$

ii) Player 2 needs to stop Player 1 from making an instant profit with his 4bet bluff of 23.5bb into a pot of 17bb, so he must 5bet with 17/(17+23.5) = 42% of his 3betting hands. In my notation,

$$\frac{B_5 + V_2}{B_5 + V_2 + B_3} = \frac{17}{17 + 23.5}.$$

iii) Player 1 needs to stop Player 2 from making an instant profit with his 5bet bluff of 88bb into a pot of 40.5bb. Player 2 uses suited aces to construct his 5bet bluffing range, which will have about 30% equity against most value ranges, so when the players get all in, Player 2 will win 30% of 201.5bb, which is about 60bb. He therefore loses 28bb of the 88bb that he 5bet jams when he's called, so this is how much he's risking to win 40.5bb. Player 1 therefore needs to call with 40.5/(40.5+28) = 59% of his 4betting range. In my notation.

$$\frac{V_1}{V_1 + B_4} = \frac{40.5}{40.5 + 28}.$$

If Player 1's complete opening range, $V_1 + B_4 + F_1$, is assumed to be known, (i) and (iii) then determine his value, bluff 4betting and folding ranges.

iv) Now that $V_1$is known, Player 2 chooses his value range, V_2, so that it has more than 50% equity against Player 1's value range.

v) Player 2 chooses his bluff 5betting range, B_5, so that Player 1's weakest value hand breaks even against his total 5betting range, $V_2 + B_5$.

If Player 1 has a 14% UTG opening range ($V_1 + B_4 + F_1 = 186$), this gives $V_1 = 34$, {QQ+, AK}, $B_4 = 22$, $F_1 = 130$. Player 2 then has a tighter value range, with $V_2 = 12$, {KK+}, $B_5 = 8$ and $B_3 = 20$. You can read the article for some advice on what hands make good 3bet and 4bet bluffing hands, and what should go into Player 2's preflop flatting range, which is treated as independent of the 3/4/5bet game. You can find similar advice in our book.

A Critique

Let's now have a close look at this argument. In the book I wrote down expressions for the amount that each player wins summed across all possible combinations of hands. Let's assume that Player 1 opens a total of T_1combos, so that we can eliminate F_1 and arrive, using Mr Janda's bet sizings, at the equivalent expressions

$$W_1 = -3.5(T_1 - V_1 - B_4)(V_2 + B_5 + B_3) + 13.5(B_4 + V_1)B_3 - 27B_4(B_5 + V_2) + V_1B_5\big(101.5\,P_{VB} - 100(1 - P_{VB})\big) + V_1V_2\big(101.5\,P_{VV} - 100(1 - P_{VV})\big),$$

and similarly for Player 2,

$W_2 = 5(T_1 - V_1 - B_4)(B_5 + B_3 + V_2) - 12(B_4 + V_1)B_3 + 28.5B_4(B_5 + V_2) + V_1B_5(101.5(1 - P_{VB}) - 100\,P_{VB}) + V_1V_2(101.5(1 - P_{VV}) - 100\,P_{VV})$.

Remember, P_{VV} and P_{VB} are Player 1's equity when he's all in with his value range against Player 2's value and 5bet bluff ranges respectively.

Let's begin with (i) above. After some rearrangement, we find that

$$W_2 = B_3(5T_1 - 17V_1 - 17B_4) + B_5(5T_1 + V_1(96.5 - 201.5P_{VB}) + 23.5B_4) + V_2(5T_1 + V_1(96.5 - 201.5P_{VV}) + 23.5B_4).$$

By the usual argument, for Player 1 to make himself unexploitable by 3bet bluffing, he has to choose his ranges so that $5T_1 - 17V_1 - 17B_4 = 0$. This gives the same answer as (i) above. Do we believe this? Let's look at Player 1's winnings, which we can write as

$$W_1 = B_3(-3.5T_1 + 17V_1 + 17B_4) + B_5(-3.5T_1 + V_1(201.5P_{VB} - 96.5) - 23.5B_4) + V_2(-3.5T_1 + V_1(201.5P_{VV} - 96.5) - 23.5B_4).$$

Doesn't this suggest that we should take $-3.5T_1 + 17V_1 + 17B_4 = 0$? The key point here is that this is *not* a zero sum game, i.e. $W_1 + W_2 \neq 0$. Since we assume that the blinds fold, there is 1.5bb in the pot before any other bets are made, so we should look carefully at the statement, 'Player 1 needs to stop Player 2 from making an instant profit with his 3bet bluff'. It seems to me that Player 1 doesn't care how much Player 2 wins; he only cares what he himself wins. A more accurate statement would therefore be, 'Player 1 needs to protect himself from 3bet bluffing'. For a zero sum game, these two statements are equivalent, but here they are not, and we actually need $-3.5T_1 + 17V_1 + 17B_4 = 0$, which means that Player 1 must 4bet with 3.5/(3.5+17) = 17% of his range.

Point (ii) is concerned with Player 2 protecting himself from 4bet bluffing. We can write

$$W_2 = B_4(-17B_3 + 23.5B_5 + 23.5V_2) + V_1(-127 + (96.5 - 201.5P_{VB})B_5 + (96.5 - 201.5P_{VV})V_2) + 5T_1(V_1 + B_5 + B_3),$$

so we need $-17B_3 + 23.5B_5 + 23.5V_2 = 0$. Player 2 must therefore 5bet with 17/(17+23.5) = 42% of his 3betting hands, in agreement with point (ii) above.

By the time we get to point (iii), where Player 1 tries to protect himself against 5bet bluffs, things are very different. Our analysis suggests that $-3.5T_1 + V_1(201.5P_{VB} - 96.5) - 23.5B_4 = 0$ is required. The expression given in point (iii) doesn't involve T_1, because it doesn't take into account the fact that Player 2's 5bet bluffs also cost Player 1 money when he folds to the original 3bet. You can't look at each successive bet in isolation. When Player 1 puts money into the pot by raising, he's already decided which hands he will fold to a 3bet, which will be 4bet/folds and which hands he will stack off with, so the money in the pot when Player 2 5bets has been committed by Player 1 in advance – in a sense, it belongs to him, not to the pot. With P_{VB}= 0.7, we find that $-3.5T_1 + 44.6V_1 - 23.5B_4 = 0$. For a 14% UTG opening range ($V_1 + B_4 + F_1 = 186$), we can therefore already deduce that $V_1 = 23$, $B_4 = 15$, which isn't far from Mr Janda's result, but doesn't map as nicely onto a sensible value betting range, and makes it hard to follow through with the equivalent of the arguments in points (iv) and (v).

Convinced? I'm not sure that I am. Saying things like 'Player 2 is protecting himself from 4bet bluffing' is really not very precise, and it's not clear to me that the alternative argument that I've just given makes any more sense than Mr Janda's. I think we should start again.

A Zero Sum Approach to the 3bet/4bet/5bet Game

If we add up each player's winnings in the game we looked at above, we find that

$$W_1 + W_2 = 1.5\, T_1(B_5 + V_2 + B_3),$$

because, between them, they hoover up the blinds whenever they play a hand. It's a key, and very restrictive, assumption that no other players become involved in the hand, and we will not change this. However, since we're restricting the problem to just two players, we should consider all the hands that Player 2 plays, i.e. we should account for the combos with which Player 2 calls, which we'll denote by C_2. We're not going to want to consider the play of the hand after Player 2 calls, but we'll have to make an assumption about the destination of the 8.5bb that's in the pot when the flop comes down. We'll assume that Player 2, who is IP, has an edge of Ebb, so that, in the long run, if he calls he gains $(0.75 + 0.5E)$bb and Player 1 gains $(0.75 - 0.5E)$bb, so that each player gets his share of the blinds, but Player 2 gains Ebb more than Player 1. We'll also assume beforehand that Player 2 plays a known total of $T_2 = B_5 + V_2 + B_3 + C_2$ combos. The values of T_1 and T_2 are determined by broader concerns of how the 3/4/5bet game fits into a total preflop strategy, and I won't try to analyse that here.

This means that now,

$$W_1 = -3.5F_1(V_2 + B_5 + B_3) + 13.5(B_4 + V_1)B_3 - 27B_4(B_5 + V_2) + V_1B_5\big(101.5\, P_{VB} - 100(1 - P_{VB})\big) + V_1V_2\big(101.5\, P_{VV} - 100(1 - P_{VV})\big) + (0.75 - 0.5E)C_2T_1,$$

$$W_2 = 5F_1(B_5 + B_3 + V_2) - 12(B_4 + V_1)B_3 + 28.5B_4(B_5 + V_2) + V_1B_5(101.5(1 - P_{VB}) - 100\, P_{VB}) + V_1V_2(101.5(1 - P_{VV}) - 100\, P_{VV}) + (0.75 + 0.5E)C_2T_1,$$

and, since

$$W_1 + W_2 = 1.5T_1T_2,$$

this is still not a zero sum game, but is a constant sum game. All we have to do next is define

$$\overline{W_1} = W_1 - (0.75 - 0.5E)T_1T_2, \qquad \overline{W_2} = W_2 - (0.75 + 0.5E)T_1T_2,$$

and *voila*, $\overline{W_1} + \overline{W_2} = 0$, and we have a zero sum game whose equilibria are the same as the original game (this is a standard trick).

Model 1

In my first attempt at studying this game, I'm going to further generalise by allowing each player to choose from five value betting strategies:

1) V = 6: {AA},
2) V = 12: {KK+},
3) V = 34: {QQ+, AK},
4) V = 56: {JJ+, AQ+},
5) V = 150: {TT+, AJ+, KQ}.

It's easy to calculate the equity of each of these ranges against the others using Equilab, and also of each range against A5s, a representative 5bet bluffing hand, so that we know P_{VV} and P_{VB} for each strategy pair. We will represent a strategy for Player 1 using the pair (v_1, B_4), where $v_1 = 1, 2, 3, 4$ or 5 according to their value betting strategy. For Player 2, we use the triple (v_2, B_3, B_5), where v_2 represents their value betting strategy. Note that, since we have fixed the total number of hands that each player is prepared to use, $F_1$and C_2 are determined by this information.

We will allow Player 1 the ten strategies (v_i, T_1) and $(v_i, 0)$, with $i = 1,2,3,4,5$ and use standard linear programming techniques to find the mixed strategy Nash equilibrium. This will be given by a linear combination of these pure strategies, thereby interpolating between zero and $T_1$and allowing for any number of 4bet bluffs, and also between each of the five value betting strategies, allowing any number of value combos. Of course, this is an approximation to the actual equity of, say, 17 value combos against 41 value combos, and doesn't allow for card removal effects, but that's something that we'll just have to bear in mind when we interpret our results. I'll try to do better in *Model 2* below. Similarly, Player 2 has the 15 strategies $(v_j, T_1, 0), (v_j, 0, T_1)$ and $(v_j, 0,0)$, with $j = 1,2,3,4,5$.

So now we have a zero sum game that we can represent in the usual way as a 10 by 15 matrix. Of course, that's quite big to analyse by hand, but a computer can find the Nash equilibrium and convert it into numbers of combos in no time at all.

Results

Let's begin by considering the equilibrium solution when Player 2 plays the same number of combos as Player 1, either by 3betting or flat calling – a not unreasonable strategy, and where Player 2's postflop edge, E, is zero. Note that the size of Player 2's calling range only affects the value of the game, not the equilibrium strategy, since T_2 doesn't actually appear in $\overline{W_1}$, as we shall see below. This makes sense, because all of the bluffing and protecting against bluffing occurs after a 3bet, not after a call (in this model). Provided that Player 2's edge isn't too large, the more hands he calls with, the better it is for Player 1, because, in the long run, he accumulates some bbs from the blinds, which can actually make the game +EV for him, at least in this model (the other players who we have neglected may have something to say about letting Players 1 and 2 run the game with wide opening ranges).

The first thing to note is that player 1 loses this game. The numbers in Figure 1 look astonishingly large, but remember that this is the total over all possible combinations. If we plot this as loss per pair of hands played ($T_1 T_2$ hands are played in total), we get Figure 2, which shows that Player 1's losses per hand are actually rather small.

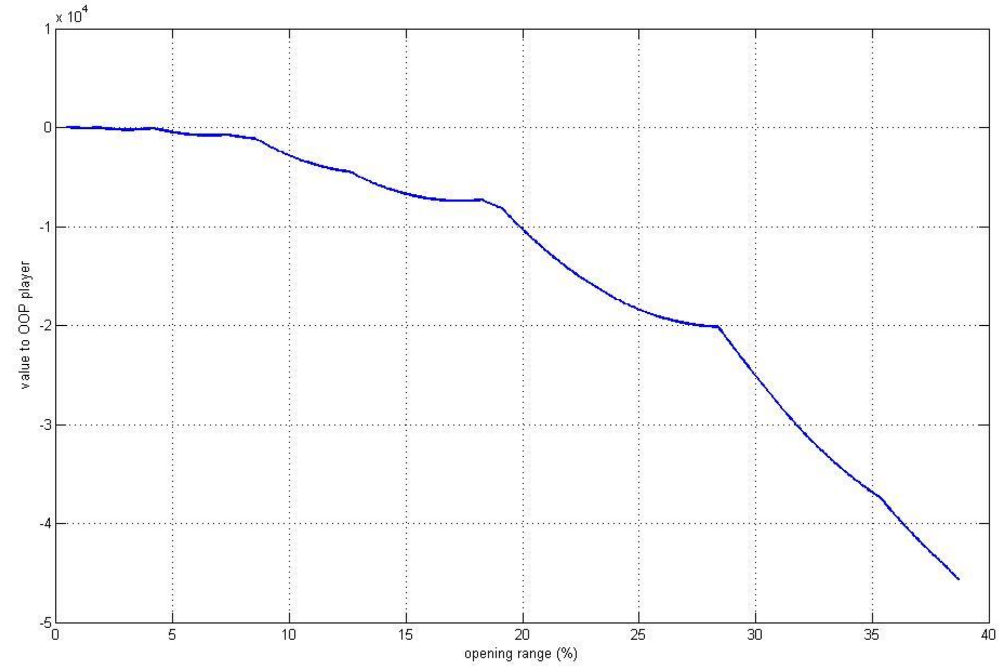

Figure 1: Total won by player 1 (negative = losses) as a function of his opening range.

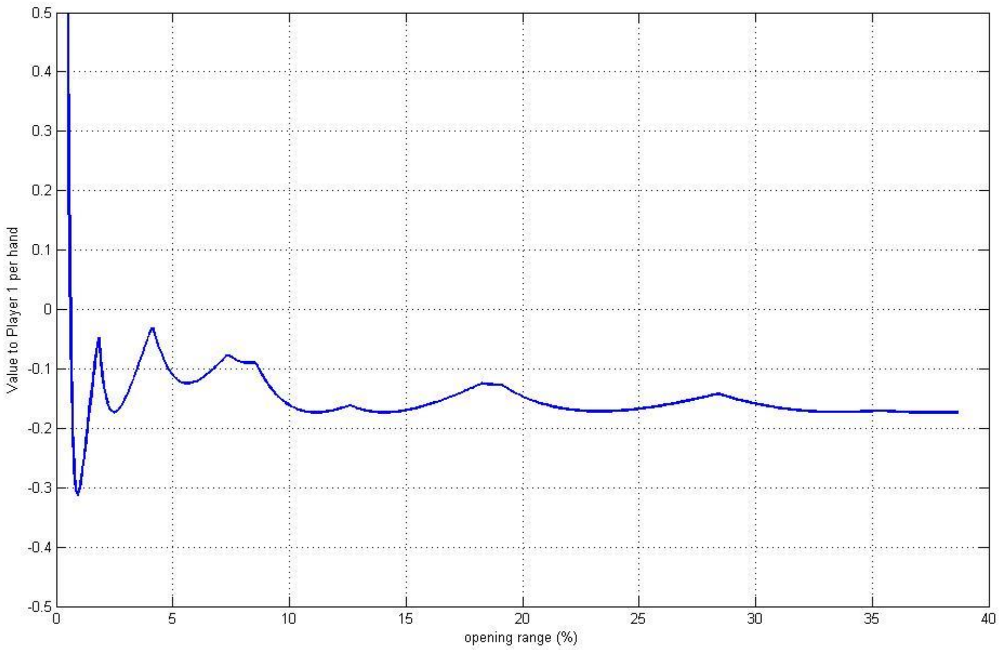

Figure 2: Amount won by Player 1, per hand played, as a function of his opening range.

Figure 3 shows Player 2's equilibrium ranges. The most striking thing about this plot is that, with the exception of a few combos for opening ranges of around 8%, the equilibrium strategy involves *no* 5bet bluffing. However, we have to take this result with a pinch of salt (remember what I said about the correct use of applied mathematics requiring some judgement?). What this means is that, for example, if Player 1's value range is QQ+ and AK, it's more +EV for Player 2 to jam, for example, QQ+ and AK, than to jam, say, KK+ and twenty two suited ace combos. Our model effectively allows Player 2 to turn AK and QQ into a bluff. In practice, on the one hand, Player 2 may want to put AK and/or QQ into his calling range, but on the other hand, does he want those 15 combos of suited Aces in his calling range? If he's not 3bet or 5bet bluffing with them and not calling with them, they're being wasted. At this point, the argument turns to the consideration of an overall preflop strategy, and falls outside the scope of this article….and if you want to read that as 'JB is a fish', feel free to do so.

Player 2's unexploitable jamming and 3bet/folding ranges get wider as his opening range gets wider, through a series of discontinuous jumps, as shown in Figure 3.

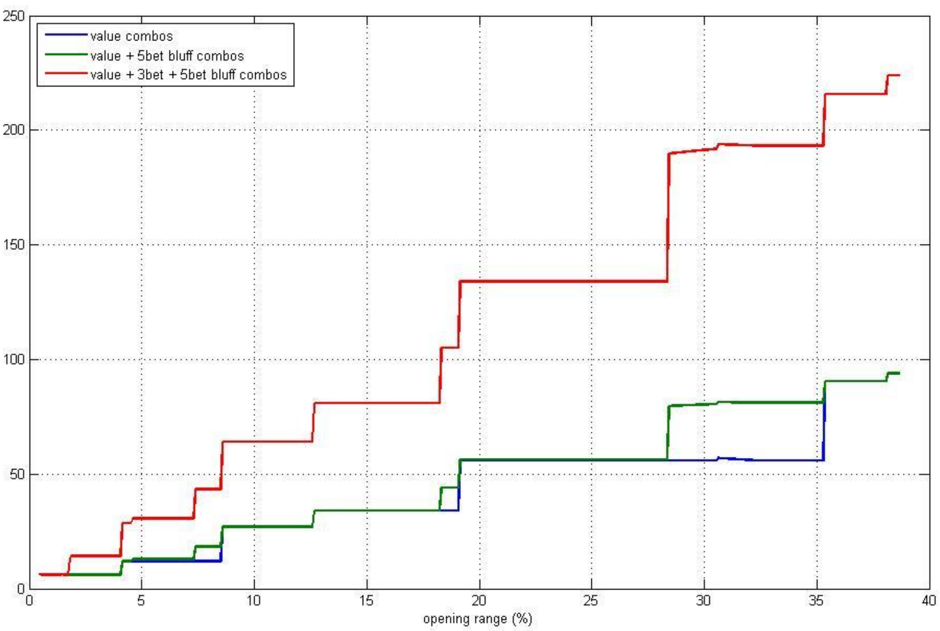

Figure 3: Player 2's value, total 5betting and total 3betting ranges as a function of his opening range.

Player 1's total 4betting range, as shown in Figure 4, is a linear function of his opening range, and his jamming range is also increasing, but not as smoothly.

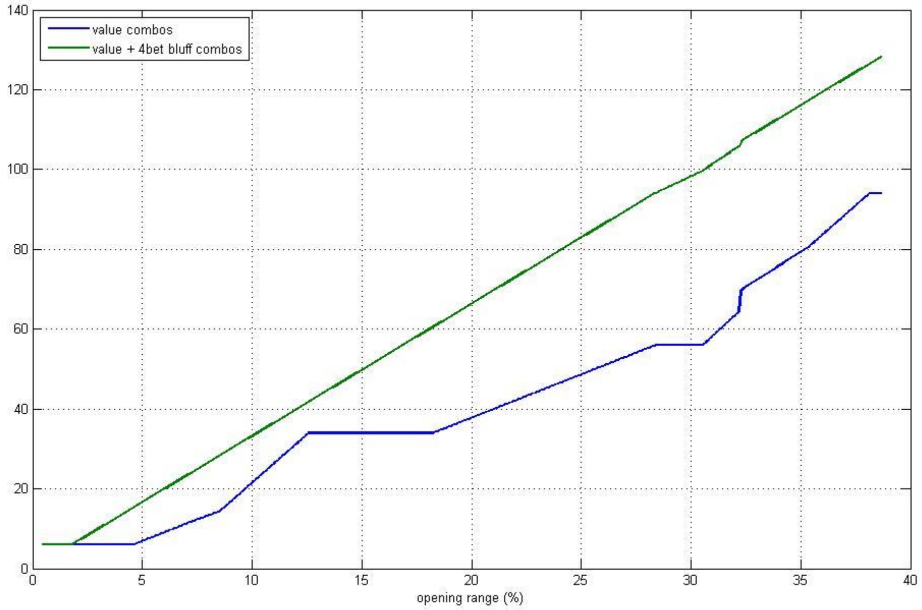

Figure 4: Player 1's value and total 4betting ranges as a function of his opening range.

We can understand a couple of features of these ranges by considering the function $\overline{W_1}$, which we can write as

$$\overline{W_1} = B_3(-(4.25 - 0.5E)T_1 + 17V_1 + 17B_4) + B_5(-(4.25 - 0.5E)T_1 + 3.5V_1 - 23.5B_4 + (201.5P_{VB} - 100)V_1) + V_2(-(4.25 - 0.5E)T_1 + 3.5V_1 - 23.5B_4 + (201.5P_{VV} - 100)V_1).$$

This suggests that, to protect against 3bet bluffing, Player 1 should take $-(4.25 - 0.5E)T_1 + 17V_1 + 17B_4 = 0$. With $E = 0$, this means 4betting 25% of his opening range, which is indeed the straight line in Figure 4. Notice that sits squarely between the 29% recommended by Mr Janda and the 19% that I suggested was correct earlier. In general, Player 1 should take

$$\frac{V_1 + B_4}{T_1} = \frac{4.25 - 0.5E}{17},$$

so that, the larger Player 2's edge postflop in single-raised pots, the less often Player 1 needs to defend against 3bet bluffing by 4betting. The slightly depressing message is 'give up more often if you're OOP against a good postflop player'.

We can also write

$$\overline{W_1} = B_4(17B_3 - 23.5B_5 - 23.5\,V_2) + V_1(17B_3 - 23.5B_5 - 23.5\,V_2 + (201.5P_{VV} - 100)V_2 + (201.5P_{VB} - 100)B_5) - (4.25 - 0.5E)T_1(B_5 + V_2 + B_3),$$

which suggests that to protect against 4bet bluffing, Player 2 should take $17B_3 - 23.5B_5 - 23.5\,V_2$ = 0, and hence jam with 17/40.5 = 42% of his 3betting range, upon which Mr Janda and I are agreed. This is indeed the case in Figure 3.

Let's finish this section by briefly looking at the effect of changing Player 2's postflop edge. In Figure 5 you can see the effect of E on Player 1's winrate per hand. It's the postflop edge that mainly determines this. The effect of playing an equilibrium strategy in 3bet pots is to avoid a large loss there. Postflop is where a good player makes his money (TT and Manu, not me obviously).

Figure 6 shows that increasing Player 2's postflop edge has the effect of allowing him to 3bet and 5bet slightly tighter ranges. Figure 7 shows that increasing Player 1's postflop edge (decreasing E), leads to him 4bet bluffing and value jamming more often, since Player 2 is 3betting and 5betting more often as his postflop edge decreases.

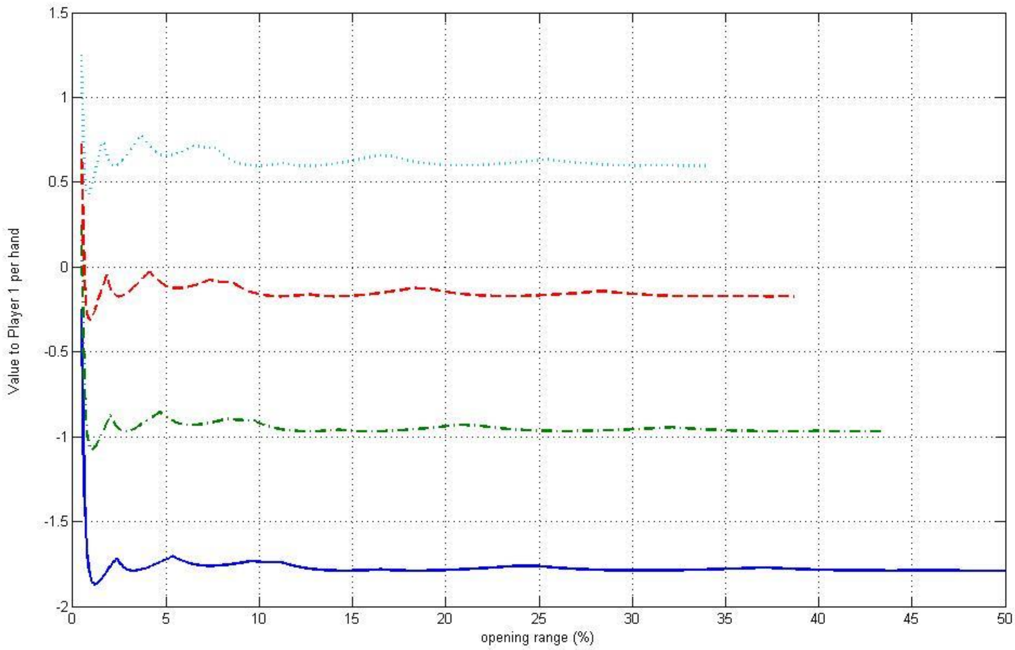

Figure 5: Value per hand to Player 1 for Player 2's postflop edge, E = 2, 1, 0 and -1.

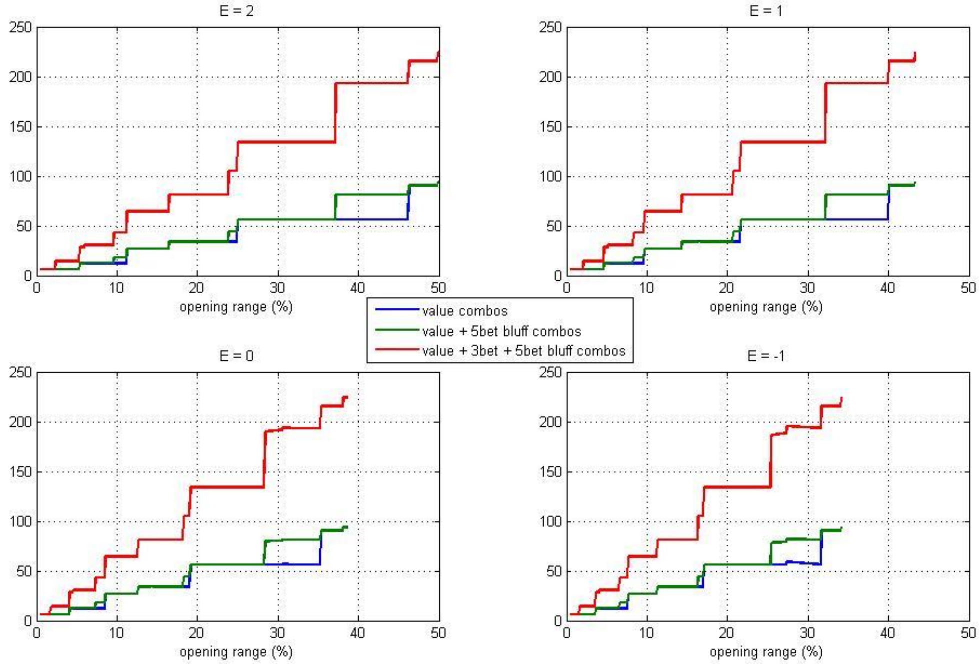

Figure 6: Player 2's equilibrium ranges for various values of E.

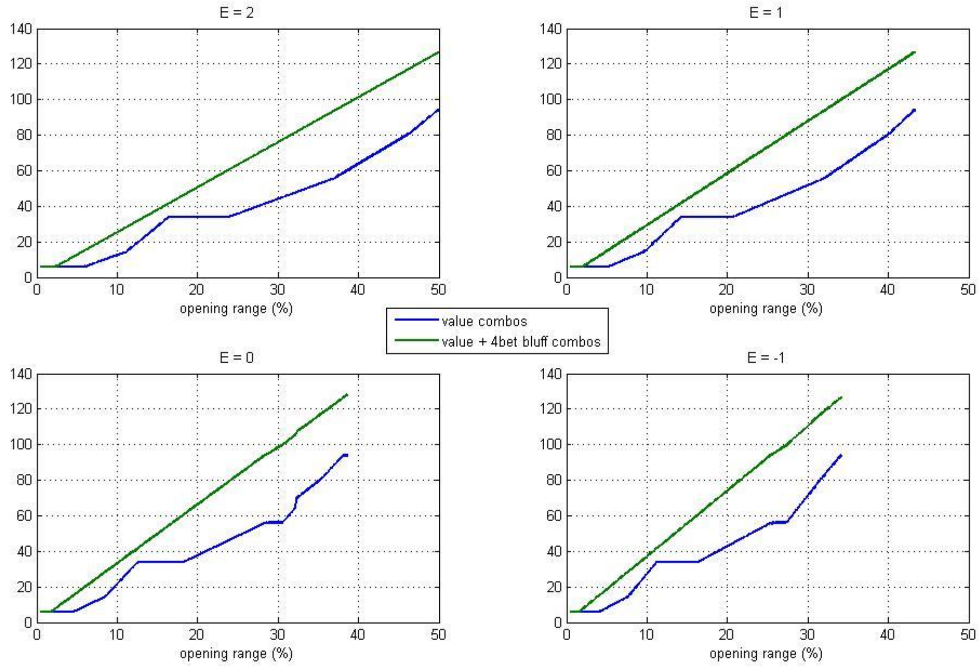

Figure 7: Player 1's equilibrium ranges for various values of E.

One thing you should bear in mind is that, as Figure 5 suggests, these equilibrium strategies are defensive, and guarantee that one player cannot exploit the other. Assuming that they're reasonable, they can be adjusted to exploit, so that, for example, if Player 1 4bet bluffs too much, Player 2 should start 5bet bluffing, and if Player 2 5bet bluffs too much, Player 1 should 4bet bluff less often, etc, etc.

Model 2

I also studied a second model, in an attempt to dissect the players' value ranges a little more carefully, and also to bring card removal effects into play.

One thing that I realized when working on this model was that ordering of Holdem starting hands in terms of strength is not a trivial exercise. I was familiar with the fact that the preflop equity of 22, AKo and 76s can't be ordered (22 > AKo > 76s > 22), so an ordering just in terms of preflop equity isn't possible and some compromise is inevitable. In terms or preflop equity against a random hand, we can order the hands as

AA>KK>QQ>JJ>TT>99>88>AKs>77>AQs>AJs>AKo>ATs>AQo>AJo>KQs>66>A9s>ATo>…

which I found quite surprising. Since we're trying to decide on an ordering of hands for sequentially putting into a preflop stacking off range, we don't really care about equity against a random hand, but rather against other strong hands. In terms of equity against the top 10% of hands the ordering is

AA>KK>QQ>JJ>AKs>AKo>TT>AQs>99>AQo>88>AJs>77>AJo>66>KQs>ATs>55>44>33>KJs>…

and against the top 25% of hands,

AA>KK>QQ>JJ>AKs>AKo>TT>AQs>AQo>AJs>99>AJo>ATs>88>ATo>77>A9s>66>KQs>A9o>A8s>…

There's also the Sklansky-Chubukov ranking, which orders the hands according to how deep stacks have to be before you can no longer profitably push your stack into a single opponent who knows what your cards are. This gives

AA>KK>AKs>QQ>AKo>JJ>AQs>TT>AQo>99>AJs>88>ATs>AJo>77>66>ATo>A9s>55>A8s>KQs>44>…

As you can see, my choice of value strategies in Model 1 is closer to the S-C ranking than any of the others.

So which is the right order in which to start adding hands as you increase your value range in the 345bet game? One thing that you can see in these lists is that, as you would expect,

AA>KK>QQ>JJ>TT>99>88>77>66>55>44>33>22

and that in the final three lists

AKs>AKo>AQs>AQo>AJs>AJo>ATs

KQs>KJs>KQo>KJo

I therefore decided to allow each player to have three separate value ranges: pairs, Aces and Kings. This gives 13 different ranges of pairs (I assume that AA is always in their value ranges), 8 of Aces and 5 of Kings (AKs and KQs are not necessarily in their value ranges), for 13x8x5 = 520 different value ranges. I also assumed that

A5s>A4s>A3s>A2s>A6s

and allowed Player 2 to add these to his stacking off range as 5bet bluffs, giving him a possible 520x7 = 3640 value strategies. I can of course also restrict these ranges, differently for each player if necessary. This is a particular issue for Player 2, who would probably prefer to have, for example, 77, AJs and KQs in his flatting range, not his 3betting range.

The problem with this new approach is that, once we start adding in the various possibilities for 3bet/folding and 4bet/folding ranges into the problem, the size of the resulting matrix gets too large for my computer to digest easily. In order to make the problem manageable, I assumed that Player 1 protects himself against 3bet bluffing by taking $-(4.25 - 0.5E)T_1 + 17V_1 + 17B_4 = 0$ and that Player 2 protects himself against 4bet bluffing by taking $17B_3 - 23.5B_5 - 23.5\,V_2 = 0$, which are the ranges that we discovered above in *Model 1*. This means that each player just has to decide which value betting strategy to use. The results that we get from this are broadly in line with those that we obtained for Model 1, but the value ranges change more smoothly because we have allowed each player more freedom to choose them. Plotted like this, it's also clear that the two players' value ranges track each other fairly closely, at least in size. As before, there's no 5bet bluffing in the equilibrium strategy, which is interesting in itself.

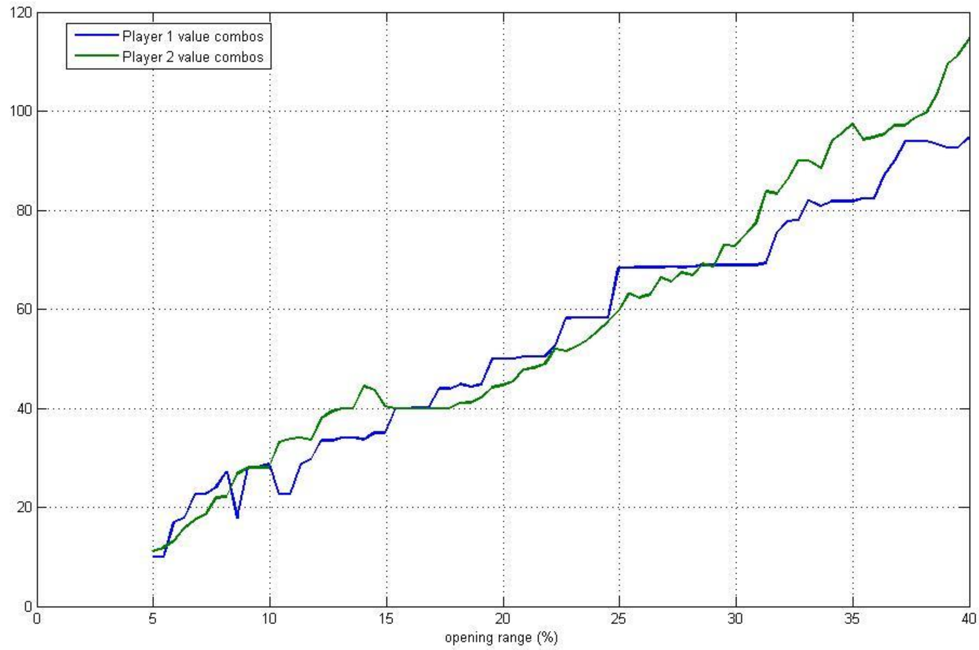

Figure 8: Model 2 - value ranges.

More interesting is the order in which the hands are added to each value range as each player's opening range gets wider. Broadly speaking, having looked at various opening ranges, the order for Player 1 is

AA>KK>AKs>{AKo,QQ}>JJ>{AQs,TT}>{AQo,99}>88>AJs>77>KQs>ATs>66>AJo,

whilst that for Player 2 is

AA>KK>AKs>{AKo,QQ}>JJ>{AQs,TT}>{AQo,99}>{ATs,KQs}>88>AJs>{KJs,77,AJo}>66, although the latter can be hard to determine, because some hands (KJs in particular) drift in and out of Player 2's value range as his overall range gets wider. There is also some dependence on Player two's edge, E, but broadly speaking, the above orderings apply and aren't far off the SC ranking.

Conclusion

So what can we learn from all of this? Well, I would hesitate to start applying all of the results above at a real 6max table, because all of this analysis assumes that only two players are involved, and in particular that the blinds always fold. I'll stick my neck out and suggest that these results are probably not too bad when the initial raise comes from early position, from someone with a fairly tight range, say 15%. In that case, each player can probably stack off with QQ+ and AK and have appropriate 3- and 4bet bluffing ranges to avoid being exploited. Once the initial raise comes from later position, the analysis above has both players raising and calling with wide ranges, and I'm sure that the presence of other players, who could probably exploit these wide ranges, must have a strong effect on the optimal strategy. In fact, the whole issue of optimal strategies in games with more than two players is a thorny one. Here's a fact to bear in mind- if a two player, zero sum game has more than one equilibrium strategy, each player wins the same amount when they each of them. This is not true in either two player nonzero sum games or in games with more than two players, such as CO v BU v SB v BB. There may be several equilibrium strategies, and each could lead to different outcomes for the players. The concept of a GTO strategy is very ill-defined in such games, yet I see poker videos and articles all over the place that talk about 6max NLHE as if it were a zero sum, two player game and that it makes sense to talk about a unique GTO strategy – it doesn't. Sadly, I don't know much about nonzero sum games, but when I've given them more thought, I will return with another article.